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# THERMAL GRAVITATIONAL CONVECTION IN A VARIABLE VECTOR FIELD OF SMALL ACCELERATIONS

V. S. Avduevskii, A. V. Korol'kov, V. S. Kuptsova, and V. V. Savichev

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The use of nearly weightless states in the manufacture of materials can allow for improvement of structure and for uniformity of mixture distribution in samples [1-3]. In the absence of gravity, small accelerations due to various perturbations play a primary role in the evolution of gravitational convection. Small accelerations are related to the rigidity characteristics of structures and are periodic in nature, where the vector for small accelerations **g** continuously changes in quantity and direction over time. In many cases, this change can to some degree of accuracy be considered as the rotation of a vector with a constant modulus and angular velocity in some fixed plane

$$|\mathbf{g}| = \mathrm{const}, \ \theta_g = \tilde{\omega}\tilde{t}, \tag{1}$$

where $\tilde{\omega}$ is the angular velocity of rotation; $\theta_g$ is the angle between the current and initial directions of the vector **g**; and the symbol ~ will be used to denote dimensional quantities.

In order to see what effect (and if there is an effect, in what manner) a change in the vector of a small local acceleration has on the evolution of convective transfer processes, we studied the model problem of thermal gravitational convection in a cylindrical volume with rotation **g** in a plane perpendicular to the axis of the cylinder.

The mathematical model for the calculation scheme is given in Fig. 1, where one must consider the transfer equations for momentum and energy in the variables T, ψ, w (the temperature, the flow function, and the vortex intensity function) and the equation for the relation between ψ and w. Using the polar coordinate system in dimensionless form with the Boussinesq approximation, these equations have the form: form the momentum transfer equation,

$$\frac{\partial w}{\partial \mathrm{Fo}} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} = \mathrm{Pr}^2\mathrm{Gr}\left\{\frac{\partial T}{\partial r}\sin(\theta - \theta_g) + \frac{\partial T}{\partial \theta}\frac{\cos(\theta - \theta_g)}{r}\right\} + \frac{\mathrm{Pr}}{r^2}\left\{r\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial \theta^2}\right\};$$

for the energy transfer equation,

$$\frac{\partial T}{\partial \mathrm{Fo}} + u\frac{\partial T}{\partial r} + \frac{v}{r}\frac{\partial T}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2},$$

$$u = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v = -\frac{\partial \psi}{\partial r},$$

and for the equation for the relation between ψ and w,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = w,$$

while the change in **g** over time is given by relation (1). Conventional definitions are used here, and the transformation to dimensionless quantities is done with the relations

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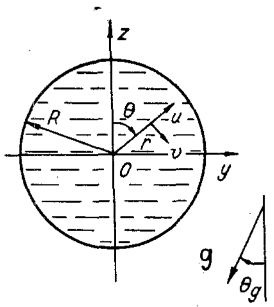


Fig. 1

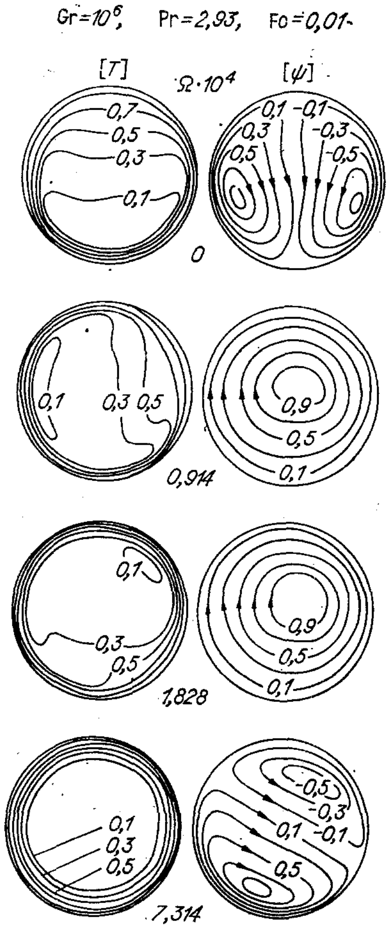


Fig. 2

$$\mathrm{Fo}=\frac{\tilde{a}\tilde{t}}{\tilde{R}^2},\quad r=\tilde{r}/\tilde{R},\quad u=\tilde{u}\tilde{R}/\tilde{a},\quad v=\tilde{v}\tilde{R}/\tilde{a},$$

$$\Delta\tilde{T}=|\tilde{T}_w-\tilde{T}_0|,\quad T=\frac{\tilde{T}-\tilde{T}_0}{\Delta\tilde{T}},\quad \psi=\tilde{\psi}/\tilde{a},\quad w=\tilde{w}\tilde{R}^2/\tilde{a},$$

$$\mathrm{Pr}=\tilde{\nu}/\tilde{a},\quad \mathrm{Gr}=\frac{|g|\tilde{\beta}\Delta\tilde{T}\tilde{R}^3}{\tilde{\nu}^2}.$$

When the specific thermal gravitational convection criteria of Grashof Gr and Prandtl Pr are supplemented by the factor $\omega$, which is the angular velocity of rotation for **g**, it is advantageous for a mathematical formulation to use a system of transfer equations that directly include this factor. For this, we will use the variables r, $\theta_t$, and Fo, where $\theta_t = \theta - \theta_g = \theta - \theta\mathrm{Fo}$ is a new tangential coordinate. Substituting the equation for w,

$$\frac{\partial w(\theta,\mathrm{Fo})}{\partial\mathrm{Fo}}=\frac{\partial w(\theta_t,\mathrm{Fo})}{\partial\mathrm{Fo}}+\frac{\partial w(\theta_t,\mathrm{Fo})}{\partial\theta_t}\frac{\partial\theta_t}{\partial\mathrm{Fo}}=\frac{\partial w(\theta_t,\mathrm{Fo})}{\partial\mathrm{Fo}}-\omega\frac{\partial w(\theta_t,\mathrm{Fo})}{\partial\theta_t}$$

into the energy transfer equation, we have

$$\frac{\partial w}{\partial\mathrm{Fo}}+u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial\theta_t}=\mathrm{Pr}^2\mathrm{Gr}\left\{\frac{\partial T}{\partial r}\sin\theta_t+\frac{\partial T}{\partial\theta_t}\frac{\cos\theta_t}{r}+\Omega\frac{\partial w}{\partial\theta_t}\right\}+\frac{\mathrm{Pr}}{r^2}\left\{r\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{\partial^2 w}{\partial\theta_t^2}\right\}. \qquad (2)$$

Here $\Omega = \tilde{\omega}\tilde{a}/|\mathbf{g}|\tilde{\beta}\Delta\tilde{T}\tilde{R}$ is a number characterizing the relation between the inertial forces arising with the rotation of **g** and the buoyant forces. In a similar way, substituting the expression for T,

$$\frac{\partial T(\theta,\mathrm{Fo})}{\partial\mathrm{Fo}}=\frac{\partial T(\theta_t,\mathrm{Fo})}{\partial\mathrm{Fo}}+\frac{\partial T(\theta_t,\mathrm{Fo})}{\partial\theta_t}\frac{\partial\theta_t}{\partial\mathrm{Fo}}=\frac{\partial T(\theta_t,\mathrm{Fo})}{\partial\mathrm{Fo}}-\omega\frac{\partial T(\theta_t,\mathrm{Fo})}{\partial\theta_t}$$

into the energy transfer equation, we have

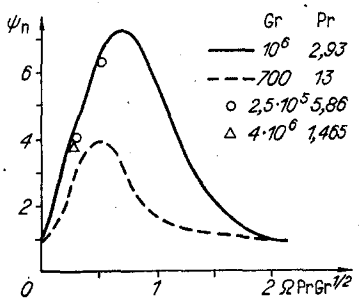


Fig. 3

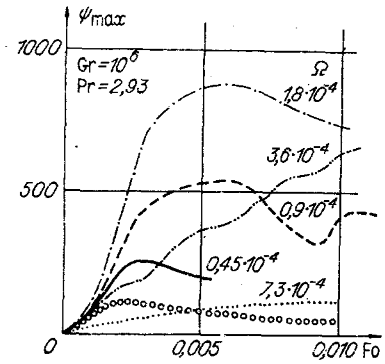


Fig. 4

$$\frac{\partial T}{\partial \mathrm{Fo}}+u\frac{\partial T}{\partial r}+\frac{v}{r}\frac{\partial T}{\partial \theta_t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta_t^2}+\Omega\,\mathrm{Gr}\,\mathrm{Pr}^2\frac{\partial^2 T}{\partial \theta_t^2}. \tag{3}$$

It is also possible to obtain the criteria $\Omega$ for rotational convection for similar transformations of the equations in a fixed coordinate system connected to **g**. Although this is possible, it is complicated for analysis since extremely cumbersome arguments are obtained for the sine and cosine functions. A more logical approach is to make the transition to the above coordinate system, since one can then explicitly separate terms in the transfer equations related to the mechanism behind rotational convection.

The equation for the relation between $\psi$ and w is unchanged since it is nonevolutional and is independent of the time Fo.

The boundary conditions have the form

$$\text{for } \mathrm{Fo}=0 \quad T=0,\ \psi=0,\ w=0,$$
$$\text{for } r=1 \quad T=1,\quad \psi=1,\quad w=\frac{\partial^2\psi}{\partial r^2}.$$

The problem is solved by the finite difference method using a modified explicit scheme with an automatic change in the difference operator [5] for the momentum and energy transfer equations and applying a passing method for the relation between $\psi$ and w.

Analyzing system (2), (3) according to the criteria $\Omega$, the following features in the evolution of thermal gravitational convection due to an angular velocity of rotation for **g** are supported by extensive experiments.

In the absence of rotation of **g**, $\Omega = 0$, and the terms that include $\Omega$ in the right-hand side of the momentum and energy transfer equations are equal to zero [$\Omega(\partial w/\partial\theta_t) = 0$, $\Omega\,\mathrm{Gr}\,\mathrm{Pr}^2\cdot\partial T/\partial\theta_t = 0$]. For a large rotational velocity for **g**, the quantity $\Omega$ goes to infinity, and the terms $\partial w/\partial\theta_t$ and $\partial T/\partial\theta_t$ go to zero to preserve the order of the quantities in the momentum and energy transfer equations. A maximum intensity of convective flow is observed for certain values of $\Omega$ that is characterized by max $\psi_{max}$ and can greatly exceed the intensity of convective flow for a fixed vector **g**. The above conclusions support the experimental results given in Figs. 2-4. If $\Omega = 7.314\cdot10^{-4}$ (Gr = $10^6$, Pr = 2.93), the isotherms take the form of concentric circles, which indicates that the derivative of the temperature in the tangential direction is equal to zero (Fig. 2).

Processing and analysis of the experimental results was done for different values of Pr, Gr, and $\Omega$, which allows one to generalize the data and to construct a simple dependence for the relative maximum flow function $\psi_n = \psi_{max}/\psi_{max,\omega=0}$ on the complex parameter $\Omega\mathrm{PrGr}^{1/2}$ (Fig. 3). This dependence indicates that the maximum intensity of convective flow exists in the range $0 < \Omega\mathrm{PrGr}^{1/2} < 2$, while for $\Omega\mathrm{PrGr}^{1/2} = 2$, the ratio $\psi_{max}/\psi_{max,\omega=0} \approx 1$, i.e., the intensity of convective flow is the same as that for a fixed vector **g**. For a further increase in the rotational velocity of g, when the indicated grouping of terms is greater than two, the intensity of convective flow is suppressed, and the ratio $\psi_{max}/\psi_{max,\omega=0}$ decreases and goes to zero in the limit.

Since for different systems (with differing physical properties, dimensions, geometries, and heat exchange conditions) the reaction to a given rotational velocity of g will vary, representing the results as a function of $\omega$ is less convenient, since calculations must be

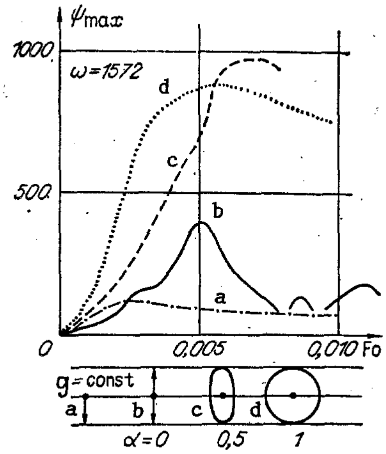


Fig. 5

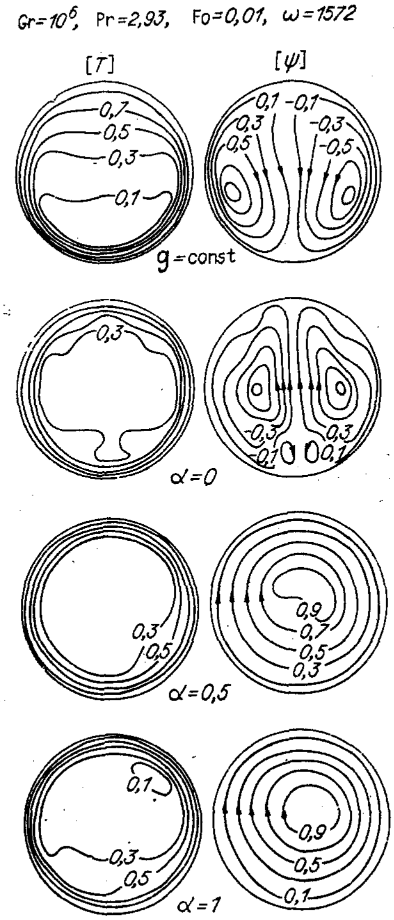


Fig. 6

done for each new system. The obtained data and the representation of functions in terms of $\Omega \mathrm{PrGr}^{1/2}$ are general in nature, since a change in the intensity of convective flow is not simply due to an increase or decrease in the rotational velocity of **g**, but is a consequence of the complex interaction between a series of processes determined by the parameters of the system and entering into the criteria Gr, Pr, and Ω. The dependence for $\psi_{max}/\psi_{max,\omega=0}$, which is given in Fig. 3, allows one for different values of Gr, Pr, and Ω to determine what effect the rotation of **g** has on the intensity of convective flow in every real case.

One should note that for different values of Ω, not only does the intensity of convective flow change, but so does the qualitative pattern of the flow function and temperature fields (see Fig. 2). In the presence of rotation of **g**, the symmetry of the fields for T and ψ is violated. If $0 < \Omega \mathrm{PrGr}^{1/2}$, the intensity of the positive vortex is greater than the case when the vector **g** is fixed, but the intensity of the negative vortex is less. The positive vortex dominates the flow function field and practically expels the negative vortex.

The observed phenomena of an increase in the intensity and suppression of convective flow for rotation of the acceleration vector are extremely important when analyzing model and numerical experiments since they allow for an approach to explaining the behavior of liquids and melted material in small acceleration fields that is based on completely new positions.

Experiments were also conducted for other types of changes in the acceleration vector

$$g_z = g_0 \cos \omega \mathrm{Fo}, \; g_y = g_0 \alpha \sin \omega \mathrm{Fo}.$$

In these cases, the dependences of $\psi_{max}$ on the dimensionless time were obtained (Fig. 5). With a decrease in the amplitude of the projection of the vector onto the axis Oy, the flow intensity drops but remains higher than would be the case in the field of a constant vector **g**. The temperature and flow function field given in Fig. 6 indicate that there are quantitative and qualitative differences in the flow for various types of space—time changes in **g** that require additional serious investigations.

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# MODELING LARGE-SCALE MIXING PROCESSES IN AN EXPANDING SUPERSONIC JET


N. A. Zheltukhin and N. M. Terekhova



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The existence of large-scale instability waves realizing large-scale mixing processes in supersonic turbulent jets is an important factor affecting both the flow structure and the noisemaking process therein. It is detected that such fluctuations in subsonic jets can result in the formation of coherent structures of the type of toruses, simple and double spirals, which under their further evolution will result in the generation of broadband noise and noise associated with the nonlinear development of instability waves [1].

Because of the technical complexity of their formulation, there are extremely few such experiments for high-velocity jets; consequently, many aspects of flow and instability wave interaction still have not been elucidated finally [2, 3]. In this situation it is impossible to underestimate the efficiency of mathematical modeling methods, which can contribute to the comprehension of definite stages in such an interaction. There have not been such researches for supersonic jets.

Speaking of the kind of large-scale waves that are evolutionary in a supersonic flow, it is necessary to note that the most important are the perturbations called the jet column mode which damp out both the mixing layer and the potential kernel during their development. As compared with the shear-layer mode originating at the root of the jet, they carry more energy, have a broad frequency spectrum, and are more characteristic for jets. The frequency and structural forms of such waves have been studied well enough [4-6].

Investigated in this paper are interaction processes of finite intensity perturbations of the jet-column-mode type with a design supersonic turbulent axisymmetric cold jet at its initial section. It is assumed that the fine-scale turbulence is in the energetic equilibrium state with the mean flow and exerts no influence on its development. There is examined what changes can occur in the stream under the action of unit waves of different spectral form (axisymmetric $n = 0$, and azimuthal or spinal $n = 1$ and 2) and more complex fluctuations of flapping type (the superposition of synchronized right- and left-twisted spirals $n = \pm 1$ and $\pm 2$).

The mean velocity vector $u = |U_0, 0, W_0|$ of such a flow has both a radial $U_0$ and a longitudinal $W_0$ component. Here and henceforth, dimensionless quantities are used, the nondimensionalization is performed by dividing by $\bar{r}$ (the initial radius), and $\bar{W}$, $\bar{\rho}$ (the longitudinal velocity and density in the flow core). In the jet core $(r < r_1 = 1 - \delta/2)u = |0, 0, 1|$, in the external field $(r > r_2 = 1 + \delta/2)u = |0, 0, 0|$, and in the mixing layer of thickness $\delta(r_1 \le r \le r_2)$ the longitudinal component is approximated by the Schlichting relationship [7]

$$W_0 = 1 - (1 - \eta^{1,5})^2,\ \eta = (1 - r + \delta/2)/\delta,\ 0 \leqslant \eta \leqslant 1.$$


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